The Elasticity of Trade for Developing Nations: Estimates and Evidence

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ABSTRACT

Quantitative results from structural gravity models of international trade depend critically on a single parameter governing the elasticity of trade with respect to trade frictions. Despite its importance, the current literature provides little evidence regarding this parameter for developing nations, which are responsible for a rising portion of world trade. We estimate this value for 129 developed and developing countries for the year 2004 using new disaggregate price and trade flow data. Our benchmark estimate for all countries is approximately 7.5 with a standard error of 0.60. We also find little evidence that the elasticity of trade differs dramatically across developed and developing nations.

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1 Introduction

Quantitative results from standard models of international trade depend critically on a single parameter governing the elasticity of trade with respect to trade frictions.[1] We provide an estimate of this elasticity consistent with heterogenous firm models of international trade for 129 countries representing 98 percent of World GDP for the year 2004 using new disaggregate price and trade flow data.

To illustrate how important this parameter is consider two examples (of many) in the empirical trade literature: Anderson and van Wincoop (2003) find the estimate of the tariff equivalent of the U.S.-Canada border varies between 48 and 19 percent depending upon the assumed elasticity of trade with respect to trade frictions. Yi (2003) points out that observed reductions in tariffs can explain almost all or none of the growth in world trade depending upon this elasticity. Thus depending upon this elasticity, the cost of the border and the growth in world trade are puzzles seeking explanations or not.

Despite its importance, evidence regarding this parameter is scarce and often comes from small samples of developed countries.[2] Estimating this parameter is difficult because standard trade models can rationalize small trade flows with either large trade frictions and small elasticities or small trade frictions and large elasticities. Thus one needs satisfactory measures of trade frictions *independent* of trade flows to estimate this elasticity. Because of the extreme data requirements, estimates of this parameter are from limited samples of developed countries. For this reason, these estimates may be appropriate for studying trade between the U.S. and Canada, but may be inappropriate for broader questions and specifically those related to developing countries which are responsible for a rising portion of world trade.

Our contribution is to provide an estimate of this elasticity for 129 countries representing 98 percent of World GDP using new disaggregate price and trade flow data. The key piece of data allowing us to estimate this elasticity is disaggregate price data from the most recent results from the International Comparison Programme. This data provides comparable prices for 129 good categories for all countries in our sample. With this data we use the maximum price difference across goods between countries as a proxy for trade frictions similar in spirit to the approach of Eaton and Kortum's (2002) study of 19 OECD countries. The maximum price difference between two countries is meaningful as it is bounded by the trade friction between the two countries via simple no arbitrage arguments. Using these proxies for trade costs from

[1] By standard models of international trade, we mean those of Krugman (1980), Anderson and van Wincoop (2003), Eaton and Kortum (2002), and Melitz (2003) as articulated in Chaney (2008), which all generate log-linear relationships between bilateral trade flows and trade frictions.

[2] See for example Head and Ries (2001) for the U.S. and Canada, Baier and Bergstrand (2001) and Eaton and Kortum (2002) for OECD countries, or the survey of these and several other studies in Anderson and van Wincoop (2004).

disaggregate price data and data on bilateral trade shares, we are able to identify and estimate the elasticity of trade with respect to trade frictions for a large cross-section of countries.

Our (preliminary) benchmark estimate for all countries is approximately 7.5 with a standard error of about 0.60. Prior research has suggested that this elasticity lies in the range between 4 and 9; see Anderson and van Wincoop's (2004) survey.[3] Our estimate provides the most comprehensive evidence regarding this parameter with it lying near the middle of the range as suggested in the literature.

We also explore if this elasticity varies with the level of development. When the sample is restricted to only high income countries, we find the estimate is only slightly lower than 7.5. When the estimate is restricted to low income countries, it is only slightly higher than 7.5. In both cases the standard errors increase to near one. Thus, we provide evidence supporting models with common elasticities of trade with respect to trade frictions across all countries.

Because our estimation approach relies heavily upon the retail prices of goods across countries, we address several important questions regarding our analysis. First, we show how our method is robust to unobserved quality differences, taxes and distribution costs within standard trade models. Second, we also estimate this elasticity in the context of a model of variable markups and non-homothetic preferences as in Simonovska (2009). Third, we provide monte carlo evidence that our approach can reliably recover this estimate in the presence of measurement error.

The remainder of the paper is organized as follows. Section 2 outlines a prototype model of international trade based on the monopolistic competition framework suggested by Melitz (2003) and Chaney (2008) and arrives at a structural equation used to estimate the elasticity parameter that resembles equations derived in very different market structures such as those employed by Eaton and Kortum (2002) and Anderson and van Wincoop (2004). Section 3 discusses the estimation procedure, while section 4 reports our estimates of the elasticity parameter using a unique data set of prices and trade flows for 129 countries in 2004. Section 5 carries out robustness analysis of the estimates of the elasticity of trade. Section 6 concludes. Finally, the appendices outline a variety of trade models and discuss the appropriate steps to estimate the elasticity parameter in these frameworks.

[3] We currently follow simple approaches to estimating this parameter as suggested in Eaton and Kortum (2002). We hope to provide monte carlo evidence regarding this approach and possibly alternative estimation approaches in future versions of this paper.

2 A Prototype Trade Model

In this section, we outline a model of trade that incorporates the monopolistic competition structure proposed by Melitz (2003) and derive a relationship mapping trade flows to trade frictions with elasticity θ—which is the key parameter of interest. Though we focus on this particular framework, we discuss how alternative frameworks such as Eaton and Kortum (2002) and Anderson and van Wincoop (2003) all generate the same exact relationship. Thus, the parameter that we ultimately estimate is not unique to one particular framework, but applicable for a broad class of models.

2.1 Consumers

Each country n has measure L_n of consumers. The maximization problem of a consumer with a unit labor supplied inelastically on the domestic labor market in country n buying varieties from (potentially) all countries $\upsilon = 1, ..., I$ is:

$$\max_{\{q_{n\upsilon}\}_{\upsilon=1}^{I} \geq 0} \left(\sum_{\upsilon=1}^{I} \int_{\Omega_{n\upsilon}} \left(q_{n\upsilon}\left(\omega\right)\right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \text{ s.t. } \sum_{\upsilon=1}^{I} \int_{\Omega_{n\upsilon}} p_{n\upsilon}(\omega) q_{n\upsilon}(\omega) d\omega \leq w_n. \quad (1)$$

$q_{n\upsilon}(\omega)$ is the quantity purchased in country n from county υ of variety ω. σ governs the elasticity of substitution across varieties ω and w_n is the wage in country n. $\Omega_{n\upsilon}$ denotes the set of varieties available to consumers in country n from country υ.

Each variety is produced by a single firm, where firms are differentiated by their productivity, ϕ, and country of origin, i. Any two firms originating from country i and producing with productivity level ϕ choose identical optimal pricing rules. This implies that we can index each variety by the productivity of its producer which we will do through out.

The demand for variety of type ϕ originating from country i consumed in a positive amount in country n, populated by L_n identical consumers, is given by:

$$q_{ni}\left(\phi\right) = w_n L_n \frac{p_{ni}(\phi)^{-\sigma}}{(P_n)^{1-\sigma}}, \quad (2)$$

where P_n is the ideal price index in this CES-based model:

$$(P_n)^{1-\sigma} = \sum_{\upsilon=1}^{I} N_{n\upsilon} \int_{\phi_{n\upsilon}^{*}}^{\infty} p_{n\upsilon}(\phi)^{1-\sigma} \mu_{n\upsilon}(\phi) d\phi. \quad (3)$$

Equations (2) and (3) characterize the equilibrium behavior from the consumers perspective.

2.2 Firms

In every country i, there exists a pool of potential entrants who pay a fixed cost, $f_i > 0$, and subsequently draw a productivity from a Pareto distribution, $T_i\phi^{-\theta}$, with support $[T_i^{1/\theta}, \infty)$. Only a measure J_i of them produce in equilibrium. Firm entry and exit drives average profits to zero. In addition, only a subset of producers, N_{ni}, with productivity draws $\phi \geq \phi^*_{ni}$, sell to a particular market n. Hence, $N_{ni} = J_i T_i/(\phi^*_{ni})^\theta$ is the measure of goods of i-origin consumed in n. Finally, we denote the density of firms originating from i conditional on selling to n by $\mu_{ni}(\phi) = \theta(\phi^*_{ni})^\theta/\phi^{\theta+1}$.

Assuming firms incur a fixed cost f_n, expensed in destination wages, in order to serve a market n, the profit maximization problem of a firm with productivity draw ϕ originating in country i and considering to sell to country n is then:

$$\pi_{ni}(\phi) = \max_{p_{ni}\geq 0} \quad p_{ni} w_n L_n \frac{p_{ni}^{-\sigma}}{(P_n)^{1-\sigma}} - \frac{\tau_{ni} w_i}{\phi} w_n L_n \frac{p_{ni}^{-\sigma}}{(P_n)^{1-\sigma}} - w_n f_n. \tag{4}$$

The optimal pricing rule of a firm with productivity draw $\phi \geq \phi^*_{ni}$ is given by:

$$p_{ni}(\phi) = \frac{\sigma}{\sigma - 1}\frac{\tau_{ni} w_i}{\phi}. \tag{5}$$

Using (5) we can derive a zero-profit condition, which determines the productivity threshold ϕ^*_{ni}:

$$\pi_{ni}(\phi^*_{ni}) = 0 \iff \phi^*_{ni} = \frac{\tau_{ni} w_i}{P_n}\left(\frac{(\sigma-1)^{1-\sigma}\sigma^\sigma f_n}{L_n}\right)^{\frac{1}{\sigma-1}} \tag{6}$$

Assuming $\theta > \sigma - 1$, define the total sales to country n by firms originating in country i as:

$$X_{ni} = N_{ni}\int_{\phi^*_{ni}}^{\infty} p_{ni}(\phi) q_{ni}(\phi) \mu_{ni}(\phi) d\phi. \tag{7}$$

In addition, the ex-ante average profits of firms originating from country i are:

$$\pi_i = \sum_{v=1}^{I} \frac{T_i}{(\phi^*_{vi})^\theta}\int_{\phi^*_{vi}}^{\infty} \pi_{vi}(\phi)\mu_{vi}(\phi) d\phi, \tag{8}$$

where potential profits from destination v are weighted by the probability that they are realized. The average profit, in turn, barely covers the fixed cost of entry:

$$w_i f_i = \sum_{v=1}^{I} \frac{T_i}{(\phi^*_{vi})^\theta}\int_{\phi^*_{vi}}^{\infty} \pi_{vi}(\phi)\mu_{vi}(\phi) d\phi. \tag{9}$$

Finally, the income of consumers from country i, spent on final goods produced domestically and abroad, becomes:

$$w_i L_i = \sum_{\upsilon=1}^{I} X_{\upsilon i}. \tag{10}$$

Using (6), (9) and (10) yield the measure of entrants in each country i:

$$J_i = \frac{L_i}{f_i} \frac{\sigma - 1}{\sigma\theta}. \tag{11}$$

2.3 Trade

Given these previous results, we now proceed to derive bilateral trade shares between countries. For concreteness, a bilateral trade share is defined to be X_{ni}/X_n where X_{ni} is the total value that country n imports from country i and X_n is the total value of consumption of all goods (either imported or domestically produced) in country n.

One can show that bilateral trade shares relative to the domestic share of consumption (or home trade share) is given by:

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \frac{\sum_{\upsilon=1}^{I} \frac{L_\upsilon T_\upsilon}{(\tau_{i\upsilon} w_\upsilon)^\theta}}{\sum_{\upsilon=1}^{I} \frac{L_\upsilon T_\upsilon}{(\tau_{n\upsilon} w_\upsilon)^\theta}} \tau_{ni}^{-\theta} \tag{12}$$

Substituting (5) into (3) and using the free entry condition defined by (6) yields the price index in country n:

$$(P_n)^{-\theta} = \sum_{\upsilon=1}^{I} \frac{J_\upsilon T_\upsilon}{(\tau_{n\upsilon} w_\upsilon)^\theta} \left(\frac{f_n}{L_n}\right)^{\frac{-\theta-1+\sigma}{\sigma-1}} \frac{\theta \sigma^{\frac{\sigma\theta+\sigma+1}{1-\sigma}} (\sigma-1)^\theta}{\theta - \sigma + 1} \tag{13}$$

Using the measure of entrants in each country i given in equation (11) and substituting it into equation (13), the relative price indices become:

$$\left(\frac{P_i}{P_n}\right)^{-\theta} = \frac{\sum_{\upsilon=1}^{I} \frac{L_\upsilon T_\upsilon}{(\tau_{i\upsilon} w_\upsilon)^\theta} \left(\frac{f_i}{L_i}\right)^{\frac{-\theta-1+\sigma}{\sigma-1}}}{\sum_{\upsilon=1}^{I} \frac{L_\upsilon T_\upsilon}{(\tau_{n\upsilon} w_\upsilon)^\theta} \left(\frac{f_n}{L_n}\right)^{\frac{-\theta-1+\sigma}{\sigma-1}}}. \tag{14}$$

Finally, we make the assumption that the fixed costs of market access are proportionate to the size of the market, $f_i = AL_i$ and the degree of proportionality is constant across countries.[4] This

[4] If one thinks of this fixed cost as a distribution/retailing cost to access a market of size L, one concludes that

assumption simplifies equation (14):

$$\left(\frac{P_i}{P_n}\right)^{-\theta} = \frac{\sum_{v=1}^{I} \frac{L_v T_v}{(\tau_{iv} w_v)^{\theta}}}{\sum_{v=1}^{I} \frac{L_v T_v}{(\tau_{nv} w_v)^{\theta}}}. \tag{15}$$

Examining equation (12) relative to equation (15), one can derive a simple expression relating trade shares to variable trade costs and aggregate relative prices.

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \tau_{ni}^{-\theta} \times \left(\frac{P_i}{P_n}\right)^{-\theta}, \tag{16}$$

2.4 Discussion

Equation (16) is the key equation of this paper and the parameter of interest is θ which governs the elasticity of trade flows with respect to trade costs.

Equation (16) is basically an arbitrage condition. If $P_n > P_i$, then country n has incentives to purchase relatively more goods from country i because they are cheaper. Or if trade costs between country n and i are large, then country n has less incentives to purchase a good from country i. Given this intuition, it should not be surprising that alternative models of international trade generate this same relationship. Below we discuss Eaton and Kortum (2002) and Anderson and van Wincoop (2003).

2.4.1 Relationship to Eaton and Kortum (2002)

Equation (16) is exactly equivalent to equation (12) derived in Eaton and Kortum (2002) which they used to estimate their elasticity. Although the two models are based on very different market structures, they turn out to deliver equivalent structural equations of θ. Mechanically, this is due to the fact θ governs the variability in the distribution of productivities in both Ricardian and monopolistic competition frameworks that employ the Fréchet and Pareto distributions, respectively. To see this, let agents consume varieties indexed by ω, where each variety is produced with efficiency $\phi \in [0, J]$. Let the measure of varieties produced with efficiency of at least ϕ be given by:

$$f(\phi; J) = J\left\{1 - \exp\left[-\frac{T}{J}\phi^{-\theta}\right]\right\} \tag{17}$$

distribution costs do not affect the estimates of θ. Alternatively, one may assume $f_i = f_e \ \forall i$, in which case a linear regression (in logs) using population data can be used. Finally, without any assumptions on the nature of fixed costs, one can use equations (14) and (16) to obtain a linear regression (in logs) and employ country-specific fixed effects to capture the presence of fixed costs.

If $J = 1$, equation (17) collapses to the Fréchet distribution used in Ricardian models. If on the other hand $J \to \infty$, (17) becomes the Pareto distribution with shape parameter θ, used in monopolistic competition models. To see this, rewrite (17) and apply the L'Hôpital rule as follows:

$$\lim_{J\to\infty} J\left\{1 - \exp\left[-\frac{T}{J}\phi^{-\theta}\right]\right\} = \lim_{J\to\infty}\left\{\exp\left[-\frac{T}{J}\phi^{-\theta}\right]\right\}\phi^{-\theta}T = \phi^{-\theta}T \tag{18}$$

2.4.2 Relationship to Anderson and van Wincoop (2003)

In principal, there is nothing unique about equation (16) to the heterogenous firm models of Melitz (2003) and Eaton and Kortum (2002). The model of Anderson and van Wincoop (2003) generates equation (16) as well. To do so, assume that each country has constant returns technologies with competitive firms producing a good which is defined by its country of origin, i.e., the Armington assumption. These assumptions imply the unit cost (and price) to deliver a country j good to country i is $p_{ij} = \tau_{ij} T_j^{\frac{1}{1-\sigma}} c_j$. Here c_j is the cost of inputs to produce one unit of the country j good and $T_j^{\frac{1}{\sigma-1}}$ is total factor productivity in country j.

Preferences are equally simple. Each country has symmetric preferences over all the (country-specific) goods with common elasticity of substitution σ. The key result from this simple model are the expenditure shares

$$X_{ij} = \frac{T_j(\tau_{ij}c_j)^{1-\sigma}}{\sum_{\ell=1}^{N} T_j(\tau_{ij}c_j)^{1-\sigma}}. \tag{19}$$

The right-hand side is country i's imports from country j divided by country i's expenditure on all traded goods. The left-hand side relates the trade cost country i faces to import a good from country j and country j's unit cost of production relative to the sum of the prices paid for imported goods.[5]

Given preferences, each country faces the following price of tradable goods for each country i

$$P_i = \Upsilon\left[\sum_{\ell=1}^{N} T_\ell \left(c_\ell \tau_{i\ell}\right)^{1-\sigma}\right]^{\frac{1}{\sigma-1}}. \tag{20}$$

Divide equation (19) with the analogous equation for country j's expenditure on country j goods and noting the relationship between the denominator of equation (19) and the price

[5] Anderson and van Wincoop (2003) call the term P_i inward multilateral resistance because it is a summary measure of the difficulty for country i to import.

index in equation (20) results in the following relationship:

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \tau_{ni}^{1-\sigma} \times \left(\frac{P_i}{P_n}\right)^{1-\sigma}. \tag{21}$$

Noticing that redefining terms in equation (21) such that $\theta = \sigma - 1$, this is the same expression as in (16) relating the bilateral trade shares to trade costs and the relative aggregate price of tradables.

3 Estimating θ

3.1 Estimation Approach

Given equation (16), our approach is to construct measures of trade shares, aggregate prices, and trade costs from data and then use equation (16) to estimate θ. Below we talk about how each was constructed. The key to our approach is how we identify trade costs from observable data *independent* of trade flows. As noted in the introduction, the difficulty in estimating θ is that τ is generally unobserved as well. Hence small trade shares can be rationalized by small trade costs and a large θ or large trade costs and small θ. The key to our approach is the utilization of a new data set of disaggregate price data that will provide a proxy for trade costs.

3.2 Data

Our sample contains 129 countries. We use trade flows and production data for the year 2004 and price data that was collected over the 2003-2005 time period to construct trade shares, aggregate prices, and proxies for trade cots.

3.2.1 Trade Shares

To construct trade shares, we used bilateral trade flows and production data in the following way:

$$\frac{X_{ni}}{X_n} = \frac{\text{Imports}_{ij}}{\text{Gross Mfg. Production}_i - \text{Total Exports}_i + \text{Imports}_i},$$

$$\frac{X_{nn}}{X_n} = 1 - \sum_{i \neq n}^{N} \frac{X_{ni}}{X_n}.$$

To construct $\frac{X_{ni}}{X_n}$, the numerator is the aggregate value of manufactured goods that country n imports from country i. Bilateral trade flow data is from UN Comtrade for the year 2004. We obtain all bilateral trade flows for our sample of 129 countries at the 4-digit SITC level. We then used concordance tables between 4-digit SITC and 3-digit ISIC codes provided by the UN and further modified by Muendler (2009).[6] We restrict our analysis to manufacturing bilateral trade flows only, namely those that correspond with manufactures as defined in ISIC Rev.2. In the denominator is gross manufacturing production minus total manufactured exports (for the whole world) plus manufactured imports (for only the sample). Put all together, this is simply computing an expenditure share by dividing the value of inputs country n imported from country i by the total value of inputs in country n. Gross manufacturing production data is the most serious data constraint we faced. We obtain manufacturing production data for 2004 from UNIDO for a large sub-sample of countries. We then used various methods to impute gross manufacturing production for countries for which data are unavailable.[7]

The home trade share $\frac{X_{nn}}{X_n}$ is simply constructed as the residual from one minus the sum of all bilateral expenditure shares.

3.2.2 Aggregate Price of Tradables

The starting point to constructing the aggregate price of tradables uses data from the 2005 round of the International Comparison Program (ICP) at the basic heading level provided by the World Bank. According to the ICP Handbook[8], unit price data on goods with identical characteristics was collected across retail locations in the participating countries during the 2003-2005 period. The lowest level of aggregation is the basic heading (BH), which represents a narrowly-defined group of goods for which expenditure data are available. There are a total of 129 BHs in the data set. Each BH contains a certain number of products. Hence, the reported price of a BH is aggregated over a narrowly-defined group of goods. An example of a basic heading is "1101111 Rice" which is made up of prices of different types of rice contained in specific packages.

To construct the aggregate price of tradables p_i, we took the geometric average across all basic heading categories that we defined as being tradable. In the Appendix we outline which goods are defined as tradable.[9] We have 62 tradable categories and are currently exploring the robustness of alternative categorizations.

[6] The trade data we obtain often reports bilateral trade flows from two sources. For example, the exports of country A to country B can appear in the UN Comtrade data as exports reported by country A or as imports reported by country B. In this case, we take the report of bilateral trade flows between countries A and B that yields higher total volume of trade across the sum of all SITC-4-digit categories.

[7] Our Data Appendix explains the approach that we used.

[8] The ICP Handbook prepared by the World Bank is available at http://go.worldbank.org/VMCB80AB40.

[9] We follow the classification of tradables used in Heston, Summers, Aten, and Nuxoll (1995).

3.2.3 Using Disaggregate Price Data to Proxy Trade Costs

To proxy trade costs τ_{ni}, we exploit *disaggregate* prices at the basic heading level across countries. To illustrate our approach, consider the following example. Suppose there are two countries (home and foreign) and two goods: TV and DVD players. Suppose the price of a TV and DVD player in the home country is 100 each, and the price of the TV and DVD player in the foreign country is 150 and 125, respectively. Trade in TV and DVD players implies the trade cost $\tau_{f,h}$ must be at least 1.50 because otherwise there would be an arbitrage opportunity.

In general, it must be the case that for a given good ℓ, $\frac{p_n(\ell)}{p_i(\ell)} \leq \tau_{ni}$, otherwise there would be an arbitrage opportunity. This implies an estimate of τ_{ni} is the maximum of relative prices over goods ℓ. To summarize, our proxy for τ_{ni}, in logs, is:

$$\log \hat{\tau}_{ni} = \max_{\ell} \left\{ \log\left(p_n(\ell)\right) - \log\left(p_i(\ell)\right) \right\}, \tag{22}$$

where the max operator is over all ℓ goods. Table 1 reports some summary statistics for the trade costs.

Table 1: Trade Cost Summary Statistics

Median	2.73	Correlation with Distance	0.16^{***}
Median High Income	2.00	Correlation with $\frac{X_{ni}/X_n}{X_{ii}/X_i}$	-0.30^{***}
		Correlation with $\frac{y_n}{y_i}$	0.49^{***}

$***$ indicates statistical difference from 0 at the 1 percent level. Correlations with both variables are expressed in logs. Number of Observations $= 10513$.

There are several points to notice. First the median trade cost for all countries corresponds with a 170 percent tariff rate equivalent. When only high income countries are considered the median trade cost declines to only 100 percent. Anderson and van Wincoop (2004) survey the literature and report that for a representative developed country, trade barriers fall in a range between 40 and 90 percent depending on the study and elasticities of substitution. Hence, the levels found here are not implausible relative to alternative approaches in the literature.

Second, the recovered trade costs correlate both with distance and the normalized trade shares $\frac{X_{ni}/X_n}{X_{ii}/X_i}$. Because the trade costs positively correlate with distance, this suggests that these costs partially reflect a known impediment to trade. Furthermore, because they correlate negatively with normalized trade shares (and stronger than distance alone) this suggests they are reflecting costs that are impeding trade.

Third, the recovered trade costs correlate strongly with relative levels of development. This result is shown by correlating the trade costs with income per worker of the importer relative to the exporter. Because the correlation is positive, this implies that the cost for a country to export is increasing the poorer the country is. Though some care should be taken with this result, it is consistent with the arguments of Waugh (2009). Furthermore, this is consistent with evidence from reduced form gravity regressions that find income per worker is an important determinant of trade flows.

4 Estimates of θ

With proxies for τ_{ni} from equation (22) we then use equation (16), bilateral trade data, and aggregate price data to estimate θ.

Table 2: Estimates of θ

	Number of Obvs. = 10513	
Approach	Est. θ	S.E.
Least Squares	7.34	0.59
Least Absolute Deviation	7.28	0.59
Method of Moments	7.74	0.62

Note: Heteroskedasticity-robust standard errors reported.

Table 2 presents the results under a variety of estimation techniques with no intercept term as the theory predicts.[10] The first two columns present estimates and standard errors of θ when all countries are considered. Least squares and least absolute deviation both produce a similar estimate around 7.34.[11] Method of moments generates similar but slightly higher estimate.

Figure 1 plots log normalized trade shares and the log trade cost multiplied by the relative price.[12] Also in this plot is the best fit line from running the appropriate regression to estimate θ, hence the slope of the best fit line is our estimate of θ. The two things to notice are that the data does exhibit a negative relationship between normalized trade shares and trade impediments. However, there is substantial noise in the data. Hence estimates from a regression without

[10]Imposing no intercept term also helps mitigate the errors in variables problem that we face. We hope to explore alternative approaches such as instrumental variables to alleviate this problem.

[11]Large deviations between least squares and least absolute deviation would suggest the estimation technique is sensitive to outliers. This does not appear to be the case, however.

[12]Because of the large amount of data, only a 20 percent random sample of the data is plotted.

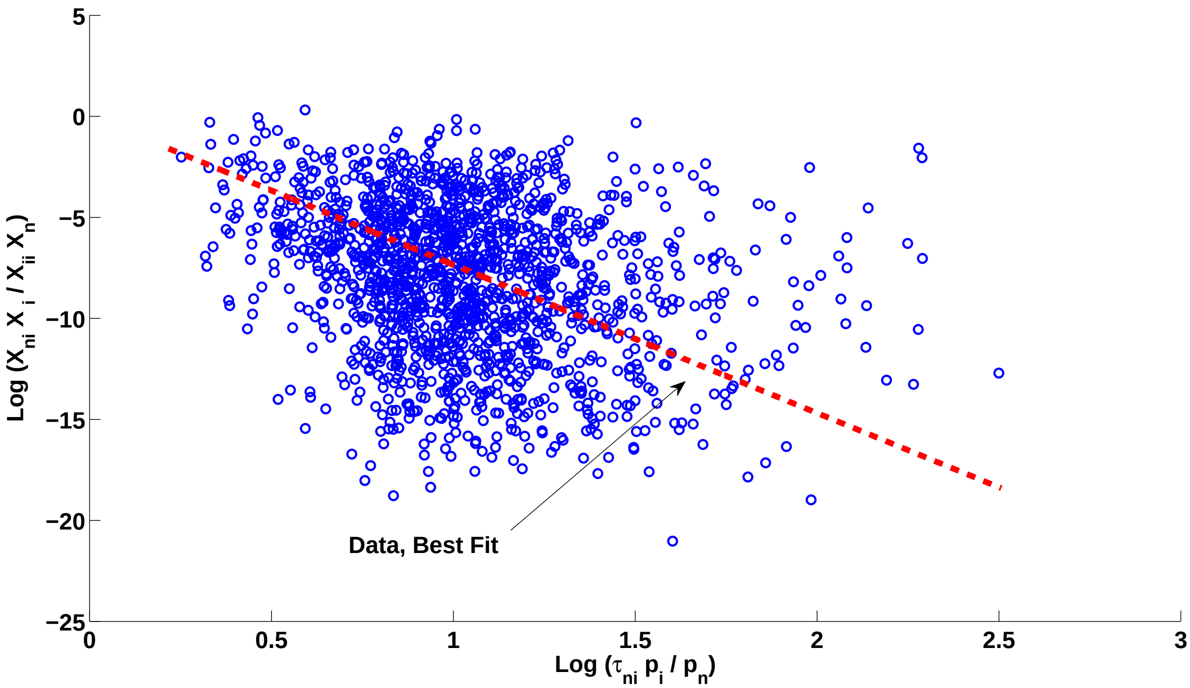

Figure 1: Plot of Log $\frac{X_{ni}/X_n}{X_{ii}/X_i}$ and Log $\tau_{ni} \times \left(\frac{p_i}{p_n}\right)$. Slope of dashed red line is the estimate of θ.

imposing a zero intercept term result in estimates of θ that are biased downwards. This is symptomatic of the errors in variables problem that we face and Figure 1 suggests this.

4.1 θ's Among Rich and Poor Nations

A key contribution of our analysis relies on the fact that we have observations on both rich and poor countries. In this section, we examine estimates of θ when only studying trade between countries of similar income level. Table 3 presents the results when the sample is restricted to only "High Income" countries and all "Non-High Income" Countries. The category "High Income" is defined by the World Bank as those countries which meet a specified level of income per capita.

The key result from this exercise is that the estimates with rich countries or with poor countries are similar and not dramatically different. The θ with only rich countries is estimated be 6.78 with a standard error of about one and the estimate for poor countries is 7.98 with a standard error slightly larger than one. While the point estimates are different, given the large standard errors, our results suggest that there are not dramatic differences in the θ's for rich countries relative to poor countries. Furthermore, these results do not appear to be sensitive relative to alternative definitions of Non-High Income countries.

Table 3: Estimates of θ Among Rich and Poor Nations

	High Income, Obvs. = 1073		Non-High Income, Obvs. = 5538	
Approach	Est. θ	S.E.	Est. θ	S.E.
Least Squares	6.78	0.92	7.93	1.17
Least Absolute Deviation	6.52	0.93	8.02	1.18
Method of Moments	7.11	0.95	8.49	1.27

Note: Heteroskedasticity-robust standard errors reported.

4.2 Results with Second Order Statistic

Eaton and Kortum (2002) use the second order statistic rater than the maximum price difference we used to proxy trade costs. In this section, we explore this alternative approach.

Their argument for using the second order statistic comes from the observation that in their price data the estimates of the trade costs were more closely correlated with normalized trade shares than the maximum. In our data set both give qualitatively similar answers, but the trade costs using maximum delivers the highest correlation with normalized trade shares and distance. The key difference that using the second order statistic does deliver is that the level of trade costs are substantially lower. For example, the median trade costs decreases to 2.24 from 2.73 when the second order statistic is used relative to the maximum.

Table 4: Estimates of θ with 2nd Order Statistic

	Est. θ	S.E.
All	9.23	0.69
High Income, Obvs. = 1073	8.54	1.15
Non-High Income, Obvs. = 5538	10.28	1.17

Note: Least Squares with no intercept used. Heteroskedasticity-robust standard errors reported.

Lower estimated trade costs results in larger estimated elasticities since normalized trade shares are kept constant. This is illustrated in Table 4 which reports the least squares estimate of θ being 9.23 and the standard error increasing to 0.69. Table 4 also presents results when only sub-samples of high and low income countries are considered. Similar to the results in the prior section, the estimates for rich nations decline slightly relative to the whole sample and the estimates for poor countries increases slightly relative to those from the whole sample. However, in both cases standard errors increase substantially. Finally, notice that our estimate of θ for high income countries using the second order statistic is very close to Eaton and Kortum's (2002) estimate of 8.28 from data on OECD countries.

5 Robustness

5.1 Models With Distribution Costs

The price data we use is collected at the retail level and thus partially reflects distribution/advertising costs. In the previous section, we showed that assuming these costs are proportional to the size of the market, they do not affect the estimates of the elasticity of trade parameter. In this section, we study a model featuring per-unit distribution costs and claim they do not affect our results at all.

Suppose firms incur a per-unit market-specific cost τ_n in order to place their product in market n.[13] Then, the production function of a firm with productivity draw ϕ originating in country i and selling to market n is $x^d(\phi) = \phi l^d/(\tau_{ni}\tau_n)$, where τ_{ni} represents the trade barrier between the country pair as in previous sections, and l^d is labor[14]. This scenario results in price indices that reflect distribution costs, while trade shares remain unchanged from the previous section. To see this, notice that the optimal price a firm with productivity draw ϕ from country i will charge in destination n becomes:

$$p^d_{ni}(\phi) = \frac{\sigma}{\sigma-1}\frac{\tau_{ni}\tau_n w_i}{\phi}. \tag{23}$$

The zero-profit condition, which determines the productivity threshold ϕ^d_{ni}, now becomes:

$$\pi^d_{ni}(\phi^d_{ni}) = 0 \iff \phi^d_{ni} = \frac{\tau_{ni}\tau_n w_i}{P^d_n}\left(\frac{(\sigma-1)^{1-\sigma}\sigma^\sigma f_n}{L_n}\right)^{\frac{1}{\sigma-1}}, \tag{24}$$

where P^d_n is the associated ideal price index.

The measure of entrants is still given by (11) since preceding steps remain unchanged. Moreover, the relative import ratios in (12) also remain unchanged since all firms serving a particular market n are assumed to incur identical per-unit distribution costs. However, the price indices

[13] This can be reinterpreted as reflecting the quality of a product sold in market n.

[14] If distribution costs are exporter- and importer-specific, they would be thought of as being discriminatory, in which case they can be interpreted as trade barriers.

now account for these distribution costs. Specifically, we can show that the aggregate price index in the model with per-unit distribution costs has the following relationship with the price index in the prototype model with no per-unit distribution costs: $P_n^d = \tau_n P_n$.

To arrive at an equation similar to (16) used to estimate θ in the benchmark case, combine (12) and $P_n^d = \tau_n P_n$ and then using the assumption that fixed costs are proportional to market size we obtain:

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \tau_{ni}^{-\theta} \left(\frac{P_i^d/\tau_i}{P_n^d/\tau_n} \right)^{-\theta} \tag{25}$$

Now notice that when we estimate τ_{ni} we want to take the maximal price difference across goods *net* the distribution cost. This value in logs equals

$$\begin{aligned} \log \hat{\tau}_{ni} &= \max_{\ell} \left\{ \log \left(p_n^d(\ell)/\tau_n \right) - \log \left(p_i^d(\ell)/\tau_i \right) \right\} \\ &= \max_{\ell} \left\{ \log \left(p_n^d(\ell) \right) - \log \left(p_i^d(\ell) \right) \right\} - \log \tau_n + \log \tau_i \end{aligned} \tag{26}$$

where d superscripts denotes a world with per-unit distribution costs.

Combining equation (25) and equation (26) in logs we arrive at the following relationship:

$$\log \left(\frac{X_{ni}/X_n}{X_{ii}/X_i} \right) = -\theta \left(\max_{\ell} \left\{ \log \left(p_n^d(\ell) \right) - \log \left(p_i^d(\ell) \right) \right\} + \log P_i^d - \log P_n^d \right) \tag{27}$$

which is the same equation as (16) but with the observable data now indicating distributional costs are included. The key implication is that under the structure of the model, distribution costs cancel out and they do not affect our results.

6 Conclusion

In this paper, we provide estimates of one of the most important and widely used parameters in the quantitative trade literature, namely the elasticity of trade with respect to trade frictions. First, we demonstrate that trade models incorporating very different market structures deliver identical structural equations relating prices, trade frictions and trade. We then employ this relationship to estimate the value of the elasticity parameter using a unique rich data set featuring highly dissagregated prices for 129 countries that account for 98 percent of World GDP in 2004. We find that the benchmark estimate of the elasticity of trade with respect to trade frictions is approximately 7.5 and that the value does not appear to vary with the level of development of a country. Moreover, the estimate is rather robust to alternative methods of analysis. Finally, the estimate is not affected by various market frictions such as distribution costs and taxes, nor does it reflect differing qualities of consumed products across countries. This makes our results applicable to a variety of quantitative models of international trade.

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A Alternative Models

A.1 Models With Sales Taxes and Mark-Ups

The price data we use, which is collected at the retail level, may also reflect sales taxes in countries that report the gross price of a good (for example a supermarket in the EU). In this section, we follow a similar argument to section 5 to show how sales taxes affect our estimates of the elasticity of trade parameter. All variables pertaining to this scenario contain the subscript t.

Suppose firms face production functions as in section 2, but consumers pay a net sales tax in destination n, $t_n - 1$[15][16]. For simplicity, we assume that tax proceeds are not rebated to consumers. Then, the consumer's problem in country n becomes:

$$\max_{\{q^{tc}_{nv}\}_{v=1}^{I}\geq 0} \quad \left(\sum_{v=1}^{I}\int_{\Omega_{nv}}\left(q^{tc}_{nv}\left(\omega\right)\right)^{\frac{\sigma-1}{\sigma}}d\omega\right)^{\frac{\sigma}{\sigma-1}} \quad s.t. \quad \sum_{v=1}^{I}\int_{\Omega_{nv}} t_n p^t_{nv}(\omega)q^{tc}_{nv}(\omega)d\omega \leq w_n. \tag{28}$$

Notice that t_n enters the budget constraint in a simple multiplicative manner. The demand for variety of type ϕ originating from country i consumed in a positive amount in country n, populated by L_n identical consumers, is given by:

$$q^t_{ni}\left(\phi\right) \;\; = \tfrac{w_n L_n}{t_n}\tfrac{p^t_{ni}(\phi)^{-\sigma}}{(P^t_n)^{1-\sigma}}, \tag{29}$$

where P^t_n is the ideal price index in country n with sales tax t_n. Using the demand function, the firm's problem becomes:

$$\pi^t_{ni}(\phi) = \max_{p^t_{ni}\geq 0} \quad p^t_{ni}\frac{w_n L_n}{t_n}\frac{(p^t_{ni})^{-\sigma}}{(P^t_n)^{1-\sigma}} - \frac{\tau_{ni}w_i}{\phi}\frac{w_n L_n}{t_n}\frac{(p^t_{ni})^{-\sigma}}{(P^t_n)^{1-\sigma}} - w_n f_n, \tag{30}$$

where it is clear that sales taxes do not affect the firm's optimal pricing rule, given by (5).

The zero-prodit condition, which determines the productivity threshold ϕ^t_{ni}, now becomes:

$$\pi^t_{ni}(\phi^t_{ni}) = 0 \iff \phi^t_{ni} = \frac{\tau_{ni}w_i}{P^t_n}\left(\frac{(\sigma-1)^{1-\sigma}\sigma^{\sigma}f_n t_n}{L_n}\right)^{\frac{1}{\sigma-1}}. \tag{31}$$

The measure of entrants is still given by (11) since preceding steps remain unchanged. Moreover, the relative import ratios also remain unchanged since all varieties sold in a particular

[15] If taxes are exporter- and importer-specific, they become discriminatory, in which case they can be interpreted as trade barriers.

[16] If firms set exogenous country-specific mark-ups, the model is identical to the one described in this section. This is not the case however if mark-ups are firm- and country-specific. We study this scenario in the following section.

market n are subject to the same per-unit sales tax. However, the price indices now account for these taxes:

$$
\begin{aligned}
&(P_n^t)^{1-\sigma} = \sum_v N_{nv}^t \int_{\phi_{nv}^t}^{\infty} \left(\frac{\sigma}{\sigma-1}\frac{\tau_{nv} w_v}{\phi}\right)^{1-\sigma} \mu_{nv}^t(\phi) d\phi \\
\Rightarrow &(P_n^t)^{1-\sigma} = \sum_v J_v T_v (\tau_{nv} w_v)^{1-\sigma} (\phi_{nv}^t)^{-\theta-1+\sigma} \frac{\theta}{\theta-\sigma+1} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \\
\Rightarrow &(P_n^t)^{1-\sigma} = \sum_v J_v T_v (\tau_{nv} w_v)^{1-\sigma} \left(\frac{\tau_{nv} w_v}{P_n^t}\left(\frac{(\sigma-1)^{1-\sigma}\sigma^{\sigma} f_n t_n}{L_n}\right)^{\frac{1}{\sigma-1}}\right)^{-\theta-1+\sigma} \frac{\theta}{\theta-\sigma+1} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \\
\Rightarrow &(P_n^t)^{-\theta} = \sum_{v=1}^{I} \frac{J_v T_v}{(\tau_{nv} w_v)^{\theta}} \left(\frac{f_n t_n}{L_n}\right)^{\frac{-\theta-1+\sigma}{\sigma-1}} \frac{\theta}{\theta-\sigma+1} \sigma^{\frac{\sigma\theta+\sigma+1}{1-\sigma}} (\sigma-1)^{\theta} \\
\Rightarrow &(P_n^t)^{-\theta} = t_n^{\frac{-\theta-1+\sigma}{\sigma-1}} \sum_{v=1}^{I} \frac{J_v T_v}{(\tau_{nv} w_v)^{\theta}} \left(\frac{f_n}{L_n}\right)^{\frac{-\theta-1+\sigma}{\sigma-1}} \frac{\theta}{\theta-\sigma+1} \sigma^{\frac{\sigma\theta+\sigma+1}{1-\sigma}} (\sigma-1)^{\theta} \\
\Rightarrow &(P_n^t)^{-\theta} = t_n^{\frac{-\theta-1+\sigma}{\sigma-1}} (P_n)^{-\theta} \\
\Rightarrow &P_n^t = t_n^{\frac{-\theta-1+\sigma}{\theta(1-\sigma)}} P_n
\end{aligned} \tag{32}
$$

To arrive at the structural relationship used to estimate θ, we combine the trade shares and (32) to obtain:

$$
\begin{aligned}
\frac{X_{ni}/X_n}{X_{ii}/X_i} &= \left(\frac{P_i^t / t_i^{\frac{-\theta-1+\sigma}{\theta(1-\sigma)}}}{P_n^t / t_n^{\frac{-\theta-1+\sigma}{\theta(1-\sigma)}}}\right)^{-\theta} \tau_{ni}^{-\theta} \left(\frac{f_n/L_n}{f_i/L_i}\right)^{\frac{-\theta-1+\sigma}{\sigma-1}} \\
\Rightarrow \frac{X_{ni}/X_n}{X_{ii}/X_i} &= \left(\frac{P_i}{P_n}\right)^{-\theta} \tau_{ni}^{-\theta} \left(\frac{f_n/L_n}{f_i/L_i}\right)^{\frac{-\theta-1+\sigma}{\sigma-1}}
\end{aligned} \tag{33}
$$

The same assumptions as in section 2 yield estimating equations for θ.

A.2 Models with Endogenous Mark-Ups

The preceding section studied how country-specific mark-ups that are common across firms affect the estimates of the elasticity parameter. In this section, we study two monopolistic competition models in which mark-ups are endogenously determined by firms. Moreover, mark-ups are good- and destination-specific and arise in response to differing price elasticities of demand across countries.

We begin with the model introduced in Simonovska (2009). All variables pertaining to this model contain a superscript s.

The maximization problem of a consumer in country n buying goods from (potentially) all countries $\upsilon = 1, ..., I$ is:

$$C_n^s = \max_{\{q_{n\upsilon}^{sc}\}_{\upsilon=1}^{I} \geq 0} \quad \sum_{\upsilon=1}^{I} \int_{\Omega_{n\upsilon}} \log(q_{n\upsilon}^{sc}(\omega) + \bar{q}) d\omega \quad s.t. \quad \sum_{\upsilon=1}^{I} \int_{\Omega_{n\upsilon}} p_{n\upsilon}^{s}(\omega) q_{n\upsilon}^{sc}(\omega) d\omega \leq w_j, \tag{34}$$

where $\bar{q} > 0$ and $\Omega_{\upsilon n}$ is compact.

The demand for variety of type ϕ originating from country i consumed in a positive amount in country n, populated by L_n identical consumers, is given by:

$$q_{ni}^{s}(\phi) \quad = L_n \left[\frac{w_n + P_n}{N_n p_{ni}^{s}(\phi)} - \bar{q} \right], \tag{35}$$

where $P_n = \sum_{\upsilon=1}^{I} N_{n\upsilon} \int_{\phi_{n\upsilon}^{s}}^{\infty} p_{n\upsilon}^{s}(\phi) \mu_{n\upsilon}^{s}(\phi) d\phi$ is an aggregate price statistic in n (not to be mistaken with the ideal price index in this economy to be derived later), and $N_n = \sum_{\upsilon=1}^{I} N_{n\upsilon}$ is the measure of varieties consumed in n.

Using the demand function, the firm's problem becomes[17]:

$$\pi_{ni}^{s}(\phi) = \max_{p_{ni}^{s} \geq 0} \quad p_{ni}^{s} L_n \left[\frac{w_n + P_n}{N_n p_{ni}^{s}} - \bar{q} \right] - \frac{\tau_{ni} w_i}{\phi} L_n \left[\frac{w_n + P_n}{N_n p_{ni}^{s}} - \bar{q} \right]. \tag{36}$$

The optimal pricing rule of such firm is given by:

$$p_{ni}^{s}(\phi) = \frac{\tau_{ni} w_i}{(\phi \phi_{ni}^{s})^{\frac{1}{2}}}, \tag{37}$$

where the productivity threshold ϕ_{ni}^{s} satisfies:

$$\phi_{ni}^{s} = \frac{\tau_{ni} w_i N_n \bar{q}}{w_n + P_n}. \tag{38}$$

Following the steps outlined in section 2 yield the following measure of entrants in this model:

$$J_i^s = \frac{L_i}{(\theta + 1) f_e}. \tag{39}$$

Moreover, the relative import shares remain unchanged. So, in order to arrive at a structural equation relating trade shares and prices, it is necessary to derive an ideal price index for the economy. Since consumer preferences are assumed to be non-homothetic, price elasticities of demand turn out to vary with the per-capita income and the (population) size of each destination. Consequently, so do mark-ups and prices firms set per destination. Since preferences

[17] In this model, the marginal utility of consuming each good is bounded at any level of consumption, thus limiting the measure of varieties that are sold in each destination. Hence, consumer demand imposes a limit on the measure of firms that serve each market, which allows to assume away the existence of market access fixed costs.

are defined for an average consumer, we derive an ideal price index, P_n^s, that exhausts the consumer's budget, namely, $P_n^s C_n^s = w_n$, where C_n^s is given in (34). Using the solution to the firm problem yields the following C_n^s[18]:

$$\begin{aligned}
C_n^s &= \sum_{\upsilon=1}^{I} N_{n\upsilon} \int_{\phi_{n\upsilon}^s}^{\infty} \log(q_{n\upsilon}^{sc}(\phi) + \bar{q}) \mu_{n\upsilon}(\phi) d\phi \\
&= \sum_{\upsilon=1}^{I} J_\upsilon \frac{T_\upsilon}{(\phi_{n\upsilon}^s)^\theta} \int_{\phi_{n\upsilon}^s}^{\infty} \log\left(\bar{q}\left(\sqrt{\frac{\phi}{\phi_{n\upsilon}^s}} - 1\right) + \bar{q}\right) \frac{\theta(\phi_{n\upsilon}^s)^\theta}{\phi^{\theta+1}} d\phi \\
&= \sum_{\upsilon=1}^{I} J_\upsilon \theta T_\upsilon \int_{\phi_{n\upsilon}^s}^{\infty} \log\left(\bar{q}\sqrt{\frac{\phi}{\phi_{n\upsilon}^s}}\right) \frac{1}{\phi^{\theta+1}} d\phi \\
&= \sum_{\upsilon=1}^{I} J_\upsilon \theta T_\upsilon \int_{\phi_{n\upsilon}^s}^{\infty} \left[\log(\bar{q}) + \frac{1}{2}\log(\phi) - \frac{1}{2}\log(\phi_{n\upsilon}^s)\right] \frac{1}{\phi^{\theta+1}} d\phi \\
&= \sum_{\upsilon=1}^{I} J_\upsilon \theta T_\upsilon \left\{\left[\log(\bar{q}) - \frac{1}{2}\log(\phi_{n\upsilon}^s)\right] \frac{1}{\theta(\phi_{n\upsilon}^s)^\theta} + \frac{1}{2}\int_{\phi_{n\upsilon}^s}^{\infty} \frac{\log(\phi)}{\phi^{\theta+1}} d\phi\right\} \\
&= \sum_{\upsilon=1}^{I} J_\upsilon \theta T_\upsilon \left\{\left[\log(\bar{q}) - \frac{1}{2}\log(\phi_{n\upsilon}^s)\right] \frac{1}{\theta(\phi_{n\upsilon}^s)^\theta} + \frac{1}{2}\frac{\theta\log(\phi_{n\upsilon}^s) + 1}{\theta^2(\phi_{n\upsilon}^s)^\theta}\right\} \\
&= \sum_{\upsilon=1}^{I} \frac{J_\upsilon T_\upsilon}{\phi_{n\upsilon}^s} \left\{\log(\bar{q}) + \frac{1}{2\theta}\right\} \\
&= \left\{\log(\bar{q}) + \frac{1}{2\theta}\right\} N_n
\end{aligned} \tag{42}$$

Using the definitions of N_n and P_n as well the productivity cutoff in (38), the measure of firms N_n becomes:

$$N_n = \left[w_n^\theta \frac{(\theta + 0.5)^\theta}{(\theta+1) f_e(\bar{q}(\theta+0.5) - \theta)^\theta} \sum_{\upsilon=1}^{I} \frac{L_\upsilon T_\upsilon}{(\tau_{n\upsilon} w_\upsilon)^\theta}\right]^{\frac{1}{\theta+1}} \tag{43}$$

[18] Deriving C_n requires to evaluate the following integral:

$$\int_{\phi_{n\upsilon}^s}^{\infty} \frac{\log(\phi)}{\phi^{\theta+1}} d\phi = \frac{\theta\log(\phi) + 1}{\theta^2(\phi)^\theta} \Big|_{\phi_{n\upsilon}^s}^{\infty} . \tag{40}$$

To ensure that the above integral is zero when evaluated at ∞, it is sufficient to assume that $\theta > 0$:

$$\lim_{\phi\to\infty} \frac{\theta\log(\phi) + 1}{\theta^2(\phi)^\theta} = \lim_{\phi\to\infty} \frac{\frac{\theta}{\phi}}{\theta^3(\phi)^{\theta-1}} = \lim_{\phi\to\infty} \frac{1}{\theta^2(\phi)^\theta} = 0, \tag{41}$$

where the L'Hopital rule has been used.

Substituting (43) into (42) yields the following ideal price index:

$$
\begin{aligned}
P_n^s &= \frac{w_n}{C_n^s} \\
&= \frac{w_n}{\left\{\log(\bar{q})+\frac{1}{2\theta}\right\}\left[w_n^\theta \frac{(\theta+0.5)^\theta}{(\theta+1)fe(\bar{q}(\theta+0.5)-\theta)^\theta}\sum_{v=1}^{I}\frac{L_v T_v}{(\tau_{nv}w_v)^\theta}\right]^{\frac{1}{\theta+1}}} \\
&= \frac{1}{\left\{\log(\bar{q})+\frac{1}{2\theta}\right\}\left[\frac{(\theta+0.5)^\theta}{(\theta+1)fe(\bar{q}(\theta+0.5)-\theta)^\theta}\right]^{\frac{1}{\theta+1}}}\left[\frac{w_n}{\sum_{v=1}^{I}\frac{L_v T_v}{(\tau_{nv}w_v)^\theta}}\right]^{\frac{1}{\theta+1}} \qquad (44)
\end{aligned}
$$

To arrive at the structural relationship used to estimate θ, we combine the trade shares and (44) to obtain:

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{P_i^s}{P_n^s}\right)^{-\theta}\tau_{ni}^{-\theta}\left(\frac{w_i/P_i}{w_n/P_n}\right) \qquad (45)$$

A few things stand out in the above equation. First, notice that (45) implies an augmented regression to estimate θ, where the term $\log\left(\frac{w_i/P_i}{w_n/P_n}\right)$ is the relative real-income ratio in the two countries and requires that a restriction be imposed in the regression to yield a coefficient on that term of exactly 1. However, the price indices are no longer proportionate to the CES ideal price index that appears in previous models. This is of course due to the non-homothetic nature of the preferences employed in this model. Moreover, one cannot expect that the price indices reported in the ICP data are computed according to this non-homothetic specification. In fact, Simonovska (2009) argues that the ICP price indices are computed using the Jevons method, discussed in Hill and Hill (2009). This method essentially combines prices of individual goods via a geometric average that is further linked across countries. So, we propose the following algorithm to estimate θ in this class of models:

(1) Make an initial guess $\theta = \theta_0$; (2) Compute the equilibrium outcomes in the calibrated model and obtain prices of individual tradable goods; (3) Aggregate goods' prices into a Jevons price index; (4) Obtain an estimate $\hat{\theta}$ using the Jevons index in (45); (5) Repeat the procedure until the initial guess and the estimated value are equivalent. Finally, compare the resulting Jevons index and the price index in (44) at the newly estimated θ to see whether there are significant differences between the two.